PRESENTATION ON DIFFERENTIAL EQUATION

CONTENTS :

- INTRODUCTION OF DIFFERENTIAL EQUATIONS.
- INVENTION OF DIFFERENTIAL EQUATIONS
- ORDER AND DEGREE OF DIFFERENTIAL EQUATIONS.
- FORMATION OF DIFFERENTIAL EQUATIONS.
- ORDINARY DIFFERENTIAL EQUATIONS (ODE).
- SOLUTION OF DIFFERENTIAL

What are Differential Equations

- Calculus, the science of rate of change, was invented by Newton in the investigation of natural phenomena.
- Many other types of systems can be modelled by writing down an equation for the rate of change of phenomena:
  - bandwidth utilisation in TCP networks.
  - acceleration of car.
  - population increase.
  - chemical change of some kind.
  - locus of a football.
- All of the above behaviour can be captured by very simple differential equations.

- A mathematical **equation** that relates a function with its derivatives is called **differential equation**,
- the function usually represent physical quantities,
- derivatives represent its rate of change
- **differential equation** defines a relationship between the two.

History of Differential Equations

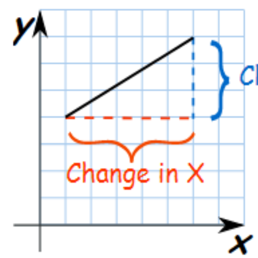
- Origin of differential equations
- Who invented idea
- Background idea.

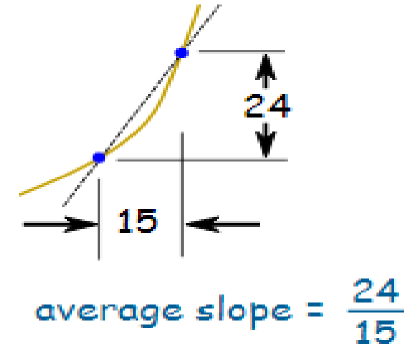
Origin of differential equations

- In mathematics history of differential equations traces the development of differential equations form calculus, itself independently invented by English physicist Isaac Newton and German mathematician Gottfried Leibniz.
- The history of the subject of differential equations in concise form a synopsis of the recent article "The History of Differential Equations 1670-1950".

Slope and Rate of change

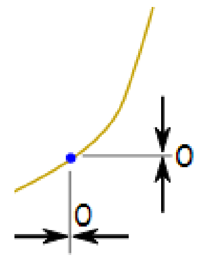
$$\text{Slope} = \frac{\text{Change in Y}}{\text{Change in X}}$$





We can find an **Average** slope between two points.

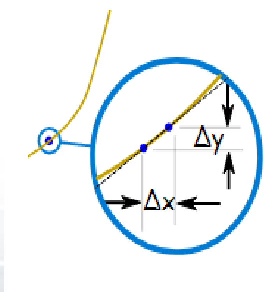
But how do we find the slope **at a point**?



There is nothing to measure!

But with derivatives we use a small difference ...

...then have it **shrink towards zero**.

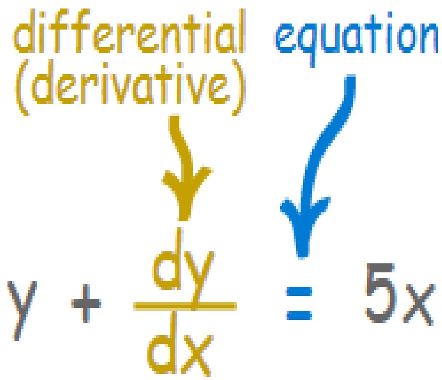


$$\text{Slope} = \frac{\text{Change in Y}}{\text{Change in X}} = \frac{\Delta y}{\Delta x}$$

Differential calculus real time video

Differential Equation

A Differential Equation is an equation with a function and one or more of its derivatives :



Example: an equation with the function **y** and its derivative $\frac{dy}{dx}$

This is a differential equation because it has 'derivative' components in it

derivative form

$$y''+y'-2y=3$$

This is a differential equation because it has 'differential' components in it

differential form

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=3$$

This is NOT a differential equation because it does not have 'differential' nor 'derivative' components in it

$$y^2+y^{-1}-2y=3$$

This is NOT a differential equation because it is not a form of equation (no 'equal' sign) even though it has 'derivative' component in it

$$y''+y'-2y$$

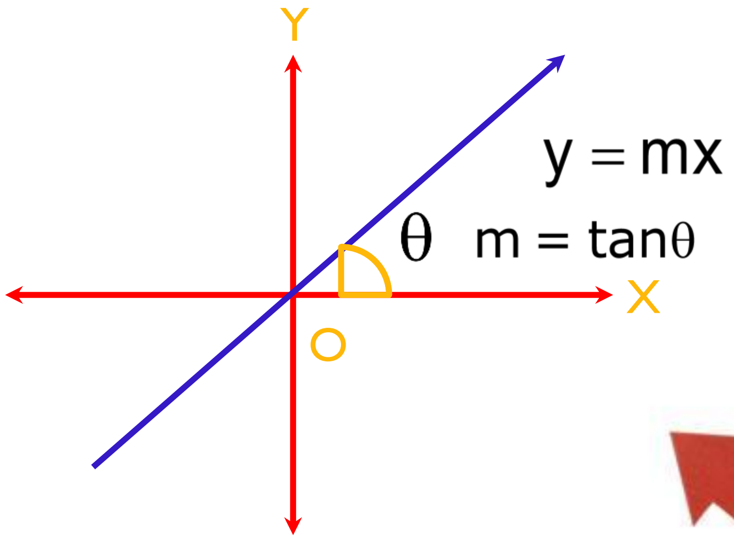
Formation of Differential Equations

the family of straight lines represented by

$$y = mx$$

$$\therefore \frac{dy}{dx} = m \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

$$\Rightarrow x\frac{dy}{dx} = y$$



is a differential equation of the first order.

Formation of Differential Equations

Assume the family of curves represented by

$$y = A\cos(x+B) \qquad \ldots (i)$$

where A and B are arbitrary constants.

$$\therefore \frac{dy}{dx} = -A\sin(x+B) \qquad \ldots (ii)$$

[Differentiating (i) w.r.t. x]

$$\text{and } \frac{d^2y}{dx^2} = -A\cos(x+B)$$

[Differentiating (ii) w.r.t. x]

Formation of Differential Equations

$$\Rightarrow \frac{d^2y}{dx^2} = -y \qquad \text{[Using (i)]}$$

$$\Rightarrow \frac{d^2y}{dx^2} + y = 0$$

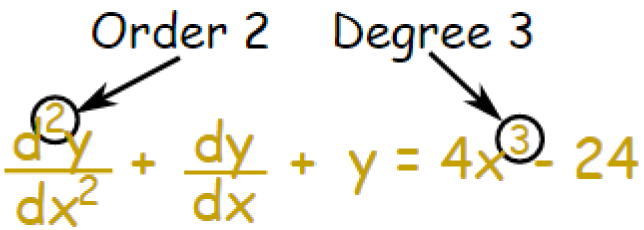
is a differential equation of second order

Similarly, by eliminating three arbitrary constants, a differential equation of third order is obtained.

Generally eliminating n arbitrary constants, a differential equation of nth order is obtained.

Order and Degree

Next we work out the Order and the Degree:



Order

The Order is the **highest derivative**

(is it a first derivative? a second derivative? etc):

Example:

$$\frac{dy}{dx} + y^2 = 5x$$

It has only the first derivative $\frac{dy}{dx}$, so is "First Order"

Example:

$$\frac{d^2y}{dx^2} + xy = \sin(x)$$

This has a second derivative $\frac{d^2y}{dx^2}$, so is "Order 2"

Example:

$$\frac{d^3y}{dx^3} + x\frac{dy}{dx} + y = e^x$$

This has a third derivative $\frac{d^3y}{dx^3}$ which outranks the $\frac{dy}{dx}$, so is "Order

Degree

The degree is the exponent of the highest derivative.

Example:

$$\left(\frac{dy}{dx}\right)^2 + y = 5x^2$$

The highest derivative is just dy/dx, and it has an exponent of 2, so this is "Second Degree"

In fact it is a **First Order Second Degree Ordinary Differential Equation**

Example:

$$\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^2 + y = 5x^2$$

The highest derivative is d^3y/dx^3, but it has no exponent (well actually an exponent of 1 which is not shown), so this is "First Degree".

(The exponent of 2 on dy/dx does not count, as it is not the highest derivative).

So it is a **Third Order First Degree Ordinary Differential Equation**

Solving :

- We **solve** it when we discover **the function y** (or set of functions y).
- There are many "tricks" to solving Differential Equations (if they can be solved!), but first: why?

Solution of a Differential Equation

The solution of a differential equation is the relation between the variables, not taking the differential coefficients, satisfying the given differential equation and containing as many arbitrary constants as its order is.

For example: $y = A\cos x - B\sin x$

$$\frac{d^2y}{dx^2} + 4y = 0$$

General Solution

If the solution of a differential equation of nth order contains n arbitrary constants, the solution is called the general solution.

$$y = A\cos x - B\sin x$$

is the general solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$

$$y = -B\sin x$$

is not the general solution as it contains one arbitrary constant.

Particular Solution

A solution obtained by giving particular values to the arbitrary constants in general solution is called particular solution.

$$y = 3\cos x - 2\sin x$$

$$\frac{d^2y}{dx^2} + y = 0.$$

is a particular solution of the differential equation .

SOLUTION OF
DIFFERENTIAL EQUATION.

Variable Separable

The first order differential equation

$$\frac{dy}{dx} = f(x,y)$$

Is called separable provided that $f(x,y)$ can be written as the product of a function of x and a function of y

Suppose we can write the above equation as

$$\frac{dy}{dx} = g(x)h(y)$$

We then say we have “separated” the variable, By taking h(y) to the LHS, the equation becomes.

$$\frac{1}{h(y)}dy = g(x)dx$$

On Integrating, we get the solution as

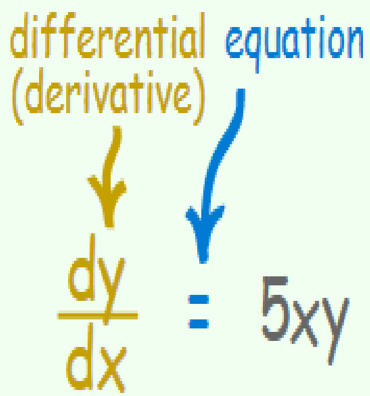
$$\int \frac{1}{h(y)}dy = \int g(x)dx + c$$

Where c is an arbitrary constant.

Separation of Variables

Separation of Variables is a special method to solve some Differential Equations

A Differential Equation is an equation with a function and one or more of its derivatives :



Example: an equation with the function **y** and its derivative $\frac{dy}{dx}$

When Can I Use it?

$$\frac{dy}{dx} = 5xy$$

$$\frac{dy}{y\,dx} = 5x$$

$$\frac{dy}{y} = 5x\,dx$$

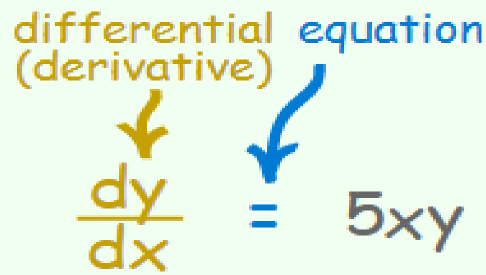
Separation of Variables can be used when:

All the y terms (including dy) can be moved to one side of the equation, and

All the x terms (including dx) to the other side.

Homogeneous Differential Equations

A Differential Equation is an equation with a function and one or more of its derivatives :



Example: an equation with the function **y** and its derivative $\frac{dy}{dx}$

Here we look at a special method for solving "Homogeneous Differential Equations"

Homogeneous Differential Equations

A first order Differential Equation is **Homogeneous** when it can be in this form:

$$\frac{dy}{dx} = F\left(\frac{y}{x}\right)$$

We can solve it using Separation of Variables but first we create a new variable $\mathbf{v} = \frac{\mathbf{y}}{\mathbf{x}}$

- $v = \frac{y}{x}$ is also $y = vx$
- And $\frac{dy}{dx} = \frac{d(vx)}{dx} = v\frac{dx}{dx} + x\frac{dv}{dx}$ (by the Product Rule)
- Which can be simplified to $\frac{dy}{dx} = v + x\frac{dv}{dx}$

Using $\mathbf{y = vx}$ and $\frac{\mathbf{dy}}{\mathbf{dx}} = \mathbf{v + x}\frac{\mathbf{dv}}{\mathbf{dx}}$ we can solve the Differential Equation.


Thank you